International Journal of Recent Research and Review, Vol. IX, Issue 2, June 2016
ISSN 2277 – 8322

Comparative Performance Analysis of Evolutionary Techniques for Designing LQR Aircraft Roll Angle Control System

Younis Shareef Dawood
College of Electronics Engineering / University of Mosul /Mosul, Iraq
E-mail: younisalshareef2015@gmail.com

Abstract- **the aircraft roll angle control system design using Linear Quadratic Regulator (LQR) is presented the LQR parameters in its performance index, which are the diagonal elements of the Q matrix, the scalar R, and the feed-forward gain N are determined using three evolutionary techniques. These techniques are mainly, the Genetic Algorithms (GA), the Particle Swarm Optimization (PSO) and lastly the artificial bee colony algorithm (ABC). Simulation results of roll controllers are presented and the performances of roll control systems are analyzed in order to minimizes, in a sequential manner, the settling time, the peak overshoot, and the maximum value of the control signal derivative. According to simulation results, it was observed that LQR-ABC roll angle control system has slightly more efficient performance than LQR-GA and LQR-PSO controllers.**
Keywords- **Aircraft Roll Angle Control System, LQR, GA, PSO ABC**.

I. INTRODUCTION

Basically, the elevator, the rudder and the ailerons are main control surfaces of an aircraft system. According to Fig.1, the elevator flaps, deflected up and down, are responsible of pitch control. The vertical tail called the rudder is responsible of yaw control .The ailerons which are flaps located outboard toward the wing tips and deflected in a differential manner, are responsible of the rolling motion of an aircraft [1].

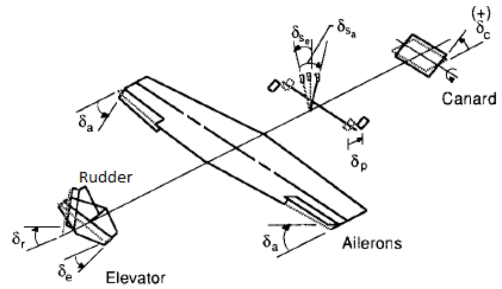

Fig. 1: Control surfaces of an aircraft system [1]

The Linear Quadratic Regulator (LQR) is one of the modern control system design tool for the roll control of an aircraft system [2]. Recently, the LQR was compared with fuzzy logic controller [3]. Other researchers used Genetic algorithm to select optimally certain LQR parameters [4]. However, design such controller requires efficient selection of the Q matrix and the scalar R in its performance index. Moreover a feed-forward gain N is used to reduce the steady state errors. Therefore, three evolutionary techniques are presented in this work to achieve this task. Mainly, the Genetic Algorithms (GA), the Particle Swarm Optimization (PSO) and the artificial bee colony algorithm (ABC) are adopted to analyze their performance in designing the LQR gain vector that satisfies certain design requirements.

II. THE DESIGNING LQR AIRCRAFT ROLL ANGLE CONTROL SYSTEM

The linearized state-space equation for roll angle of an aircraft system is given by:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \qquad (1)$$

Where, A is the plant matrix, B is input column vector, C is output raw vector , and $x(t)$ is the state vector. Numerically, [5]:

$$\begin{bmatrix}\Delta\dot{\beta}\\ \Delta\dot{P}\\ \Delta\dot{r}\\ \Delta\dot{\phi}\end{bmatrix}=\begin{bmatrix}-0.254 & 0 & -1 & 0.184\\ -15.969 & -8.395 & 2.19 & 0\\ 4.549 & -0.349 & -0.76 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}\Delta\beta\\ \Delta P\\ \Delta r\\ \Delta\phi\end{bmatrix}+\begin{bmatrix}0\\ -28.916\\ -0.224\\ 0\end{bmatrix}\Delta\delta_a \qquad (2)$$

$$y=\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta P\\ \Delta r\\ \Delta\phi\end{bmatrix}$$

Where, $\Delta\beta$ is the sideslip angle, P is the angular rate component of roll axis, r is the angular rate component of yaw axis, φ is the orientation of aircraft (roll angle) in the earth-axis system δ_a is the aileron deflection angle.

The Linear quadratic regulator (LQR) is based on the manipulation of the state-space equations of motion and utilizing a systematic procedure in the design process. lateral directional control of an aircraft with LQR feedback controller is shown in fig. 4 [5]. The Control law is given as:

$$u(t)=-K.x(t)+\Delta\delta_a\ . \qquad (3)$$

Such control signal is used to minimize the performance index

$$J=\int_0^\infty (x^T Q\, x+u^T R u)\ dt \qquad (4)$$

In Equation (3) and (4), K is the feedback gain matrix, Q is state-weight-performance matrix and R is input-weight value, and N is a feed-forward gain used to reduce steady-state error, as shown in Fig. 2.

The value of the feedback gain matrix, K, has been designed using the **lqr** MATLAB function with R=1,N=8.86603 and $Q=qC'C$, where q is the weighting factor set to be 75. The gain matrix is found to be $K=[0.5284,-0.5349,-0.0917,-8.6567]$ [5]

III. THE EVOLUTIONARY TECHNIQUES

The Evolutionary Techniques (ETs) are an adaptive search optimization techniques based on natural phenomena mechanisms. They have been utilized as techniques to solve sub-optimal problems (optimal within a predetermined hyper space). The (ETs) start with an initial random population (individuals) that represent a solution of the problem .The performance is evaluated by a fitness function. Basically, The (ETs) consist of main stages (operations) that allow , in current generation, the creation of new population (individuals) which may satisfy the fitness function better than the previous generation. This algorithm is running for many generations (iterations) and terminated if the fitness function is satisfied with individuals that represent the required solution to the problem within the predetermined search space.

Here in this work, three (ETs) are presented which are the Genetic Algorithms (GA) , the Particle Swarm Optimization (PSO) and the Artificial Bee Colony algorithm (ABC) . In these (ETs), the population (individuals) is the diagonal elements in Q matrix (q_{11} q_{22} q_{33} q_{44}), the scalar value R and the feed-forward scaling factor N.

The fitness (objective) function is the measure of the quality of the population (individuals) in meeting the design requirements. The following fitness function is proposed to get less settling time, fewer peaks overshot and less harmful control signal in comparison to that obtained in [5].

Hence, minimize $J_{Fitness}$

Where,

$$J_{Fitness}=\sum_{i=1}^{3} w_i\left|e_i\right|\le\varepsilon \qquad (5)$$

Where, $\left|e_1\right|=\left|\dfrac{t_{s\ LQR-ET_k}}{t_{s\ LQR}}\right|$, $\left|e_2\right|=\left|\dfrac{Mp_{LQR-ET_k}}{Mp_{LQR}}\right|$,

$\left|e_3\right|=\left|\dfrac{\max(\dot{u})_{LQR-ET_k}}{\max(\dot{u})_{LQR}}\right|$, $\varepsilon=0.01$, In Equation (5), w is the weight for each specific design requirements, t_s is the settling time , Mp is the maximum over shoot , $\max(\dot{u})$ is the maximum value of the derivative of the control signal u, and ET_k is the k_{th} Evolutionary Technique (ET_1 is the GA, ET_2 is the PSO , and ET_3 is the ABC).

The above fitness (objective) function is accomplished in a sequential manner as follows:

1. For the trial i
2. Set $J_{Fitness} = 0.8|e_1| + 0.1|e_2| + 0.1|e_3|$
3. Apply the corresponding ET (GA, or PSO, or ABC).
4. Select the best 30 individuals that have minimum fitness function. Set the rank of each individual as (rank= r).
5. Set $J_{Fitness} = 0.1|e_1| + 0.8|e_2| + 0.1|e_3|$.
6. Apply the corresponding ET (GA, or PSO, or ABC).
7. Select the best 30 individuals that have minimum fitness function. Increase the rank for those individuals that selected again in this step as (rank=r+1).
8. Set $J_{Fitness} = 0.1|e_1| + 0.1|e_2| + 0.8|e_3|$.
9. Apply the corresponding ET (GA, or PSO, or ABC).
10. Select the best 30 individuals that have minimum fitness function. Increase the rank for those individuals that selected again in this step as (rank=r+2).
11. Set trail as (i+1) . If trial = *max_trial* then select the individual that has maximum rank as the optimal solution, else back to step 1.

The Genetic Algorithms (GA)

The main steps of the GA can be summarized as follow [6]:

1. Initialize randomly a population of strings with individuals that represent the unknown variables within predetermined search space.
2. Evaluate the fitness for each string according to Eq.(5).
3. Perform the selection, crossover, and mutation on this population to create the next generation.
4. Stop if the stopping conditions are fulfilled, else, return to step 1.

A. The Particle Swarm Optimization (PSO)

The executive steps of parameter selection based on PSO are as follows [7]:

1. Initialization of particle populations. Iteration times and particle number are given. The values of the diagonal elements in Q matrix (q_{11} q_{22} q_{33} q_{44}), the scalar value R and the feed-forward scaling factor N which are particle's position x_i and $v_i = [v_1, v_2, v_3, v_4, v_5]$ are velocity, and are generated randomly. Evaluating the fitness function $J_{fitness}$ according to Equation (5).
2. If the particle's current fitness having less $J_{fitness}$ than the previous P_{best} , the current fitness value is regarded as current P_{best} ; if the particle's current fitness having less $J_{fitness}$ than global g_{best} , the current fitness value is regarded as current g_{best} . Update the velocity and position of the particle iteratively according to:

$$v_{i+1} = v_i w_i + c_1 * rand() rand() \\ * (p_{best} - x_i) + c_2 * \; * (g_{best} - x_i) \quad (6)$$

and $x_{i+1} = x_i + v_{i+1}$ (7)

Where, $w_i \in [0,1]$ is the inertia coefficient of i_{th} particle ; c1 and c2 are learning rates which are non-negative constants; rand() is randomly numbers between 0 and 1 ; x_i and v_i are the position and the velocity of the particle, respectively. P_{best} is the position's best fitness found so far for the i_{th} particle, and g_{best} is the best neighborhood position.

3. If the algorithm meets $J_{Fitness}$ or reaches maximum iterations number as explained in the GA, the algorithm is stopped, otherwise go back to step 2.

B. *The Artificial Bee Colony (ABC) algorithm*

The main executive steps of the ABC technique are [8]:

1. Initialize population with random solutions (or the values q_{11} , q_{22},q_{33},q_{44} ,R ,and N).
2. Evaluate fitness of the population ((Equation 5).
3. While (stopping criterion not met)//Forming new population.
4. Select sites for neighborhood search. The bees that have the highest fitnesses are chosen as "selected bees" and sites visited by them are chosen for neighborhood search.
5. Recruit bees for selected sites (more bees for best e sites) and evaluate fitnesses. Here, the algorithm conducts searches in the neighborhood of the selected sites, assigning more bees to search near to the best e sites. The bees can be chosen directly according to the fitnesses associated with the sites they are visiting. Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighborhood of the best e sites which represent more favorable solutions are made richer by recruiting more bees (colony's explorers) to follow them than the other selected bees. In the artificial bees, the artificial scouts may have the quick feasible solutions as a task. Together with scouting, this differential recruitment is a key operation of the Bees Algorithm.
6. Select the fittest bee from each patch that have the highest fitness in order to form the next bee population.
7. Randomly assign remaining bees to search around the search space scouting for new promising solutions and evaluate their fitnesses.
8. End While.

IV. SIMULATION AND RESULTS

The LQR is designed for the roll sliding of flight control system. The GA, PSO and ABC techniques are used to fully utilize the tuning factors embedded in the LQR which are mainly the diagonal elements in Q matrix (q_{11} q_{22} q_{33} q_{44}), the scalar value R and the feed-forward scaling factor. The upper and the lower values of these tuning factors are chosen to be $0 < q_{ii} \leq 200 \quad (i = 1,2,3,4) \quad ,0 < R \leq 5$ $,and \quad 0 < N \leq 10$.

Table (1) presents the main parameters of the three ETs used in this study.

Table I

The main parameters of the three ETs

The ETs	Value/Method
GA	Population size =100, Maximum number of generation=400, Normalized Geometric Selection with probability 0.05, scattering crossover with probability 0.2, Uniform Mutation with probability 0.01.
PSO	c_1 *and* c_2 = 1.5, particle populations = 80,particle number =6, iterations times=100.
ABC	Population size =100, the colony size is 25,the control factor to leave the food source is 120, the number of run is 3.

For comparison purposes, the results of the LQR obtained by [5] were reproduced .Hence, the output, the input, and the states responses are illustrated in Figures (3),(4) and (5) , respectively. The similar responses for the LQR tuned further by GA (LQR-GA) are presented in Figures (6), (7) and (8). Moreover, the responses for the LQR further tuned by PSO (LQR-PSO) are presented in Figures (9), (10) and (11), while those responses for the LQR further tuned by ABC (LQR-ABC) are presented in Figures (12),(13) and (14).

The numerical values that indicating the performance of each proposed design technique together are presented in Table II with the resultant tuning parameters

Table II
The resultant tuning parameters of the three ETs

The Design Technique	Q-matrix	R	N	t_s Sec.	M_p %	$\max(\dot{u})$ rad/Sec.
LQR-[5]	Diagonal [0 0 0 75]	1	8.66	0.54	2.8	1.68
LQR-GA	Diagonal [0.05 1.1 0.04 72]	2.1	7.8	0.65	0	1.32
LQR-PSO	Diagonal [0.03 0.96 0.01 76.5]	1.96	8.2	0.62	0	1.42
LQR-ABC	Diagonal [0.01 1 0.01 75.25]	2.01	8	0.62	0	1.38

V. CONCLUSIONS

A comparative performance study of three evolutionary techniques, mainly the Genetic Algorithms (GA), the Particle Swarm Optimization (PSO) and the artificial bee colony algorithm (ABC), have been presented. The three techniques are used for selecting the LQR parameters, which are the diagonal elements of the Q matrix, the scalar R in its performance index, and the feed-forward gain N used to reduce the steady state errors. The same fitness (objective) function is used in the three evolutionary techniques that minimize in a sequential manner the settling time, the peak overshoot, and the maximum value of the control signal derivative. One can clearly notice the superiority of the LQR-ABC, LQR-PSO, and LQR-GA in comparison to that in [5]. Moreover, the LQR parameters selected by the evolutionary techniques reduces the angular rate component of roll axis ($X_2(t)$) in comparison to that obtained in [5]. Finally, it is found that the capability of the ABC in further tuning of the LQR is slightly more efficient than the PSO and the GA to meet the same design performance criterion.

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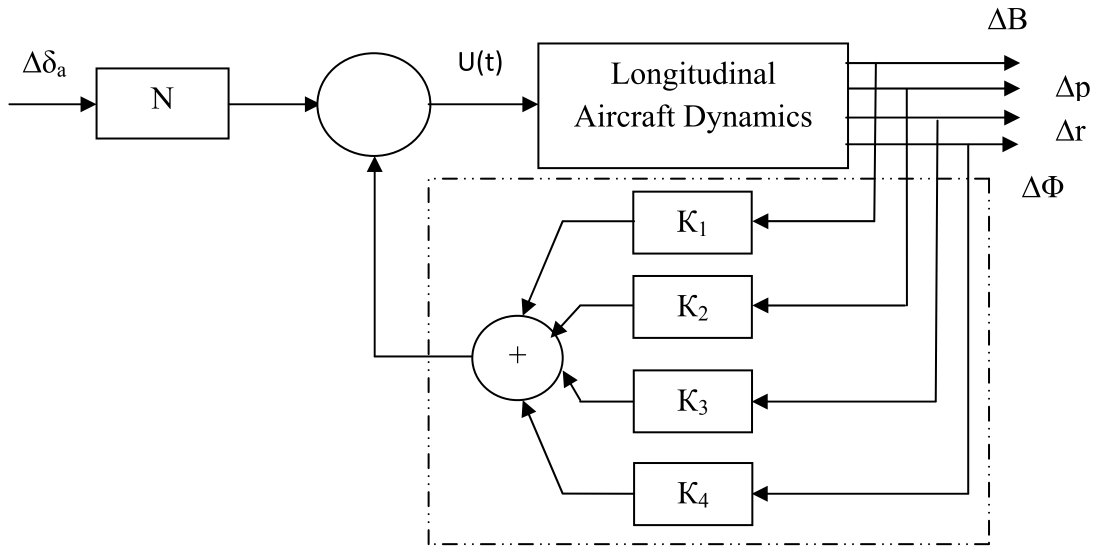

Fig. 2: Roll Control system with full-State Feedback Controller of Reference Input

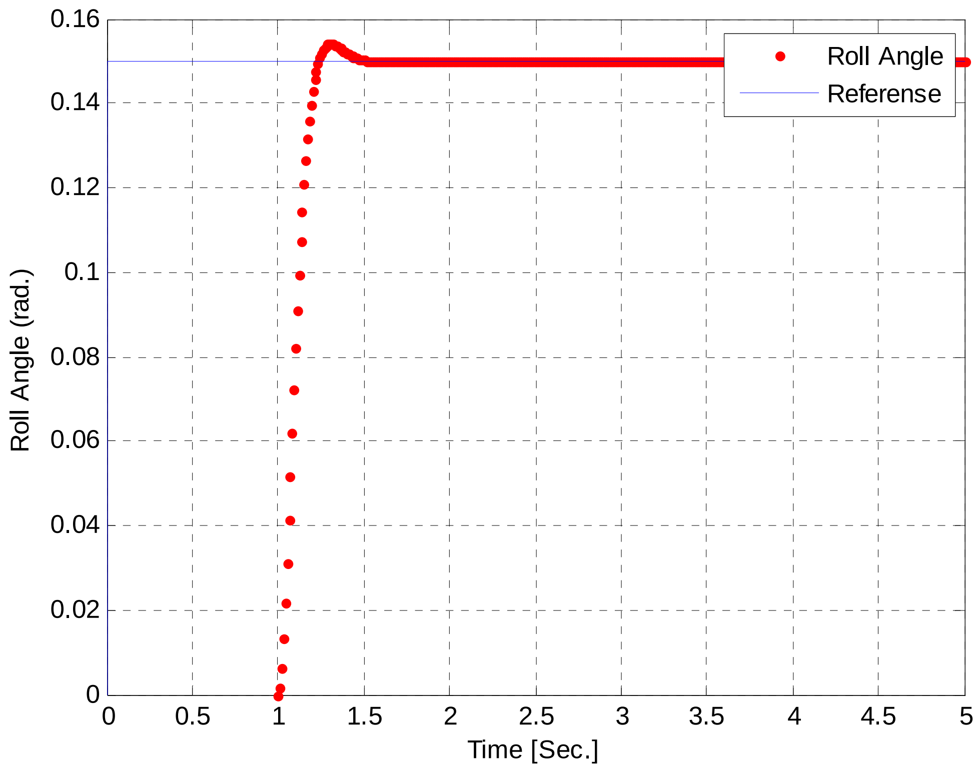

Fig. 3 : Step response of the roll angle for the case of LQR of [5]

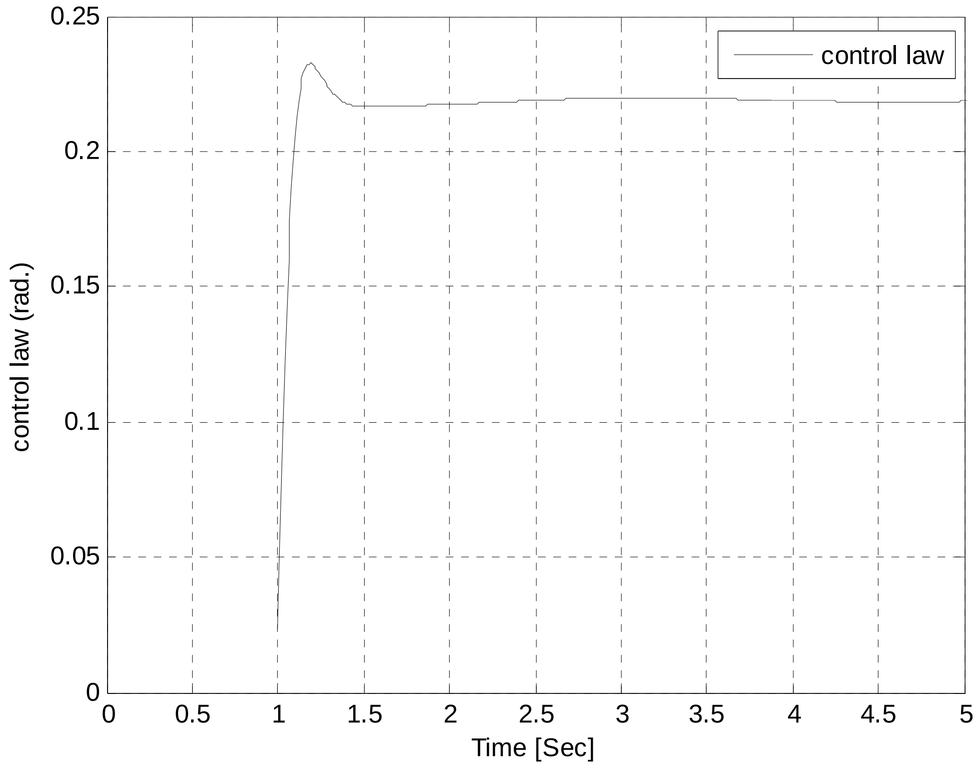

Fig. 4 : The response of the control signal for the case of LQR of [5]

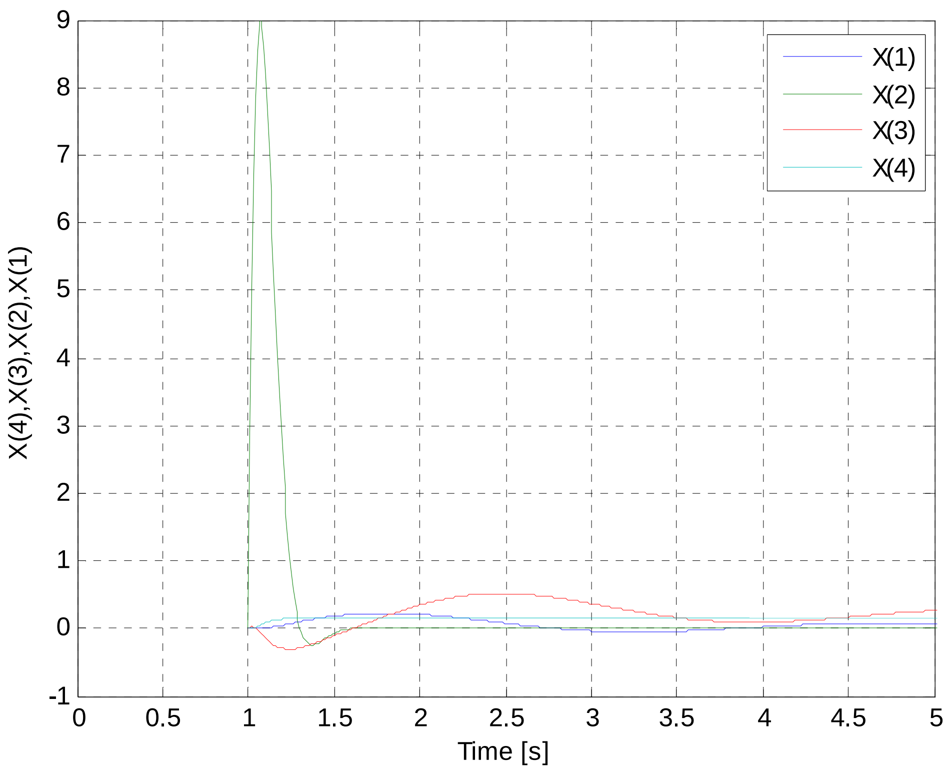

Fig. 5 : The response of aircraft states for the case of LQR of [5]

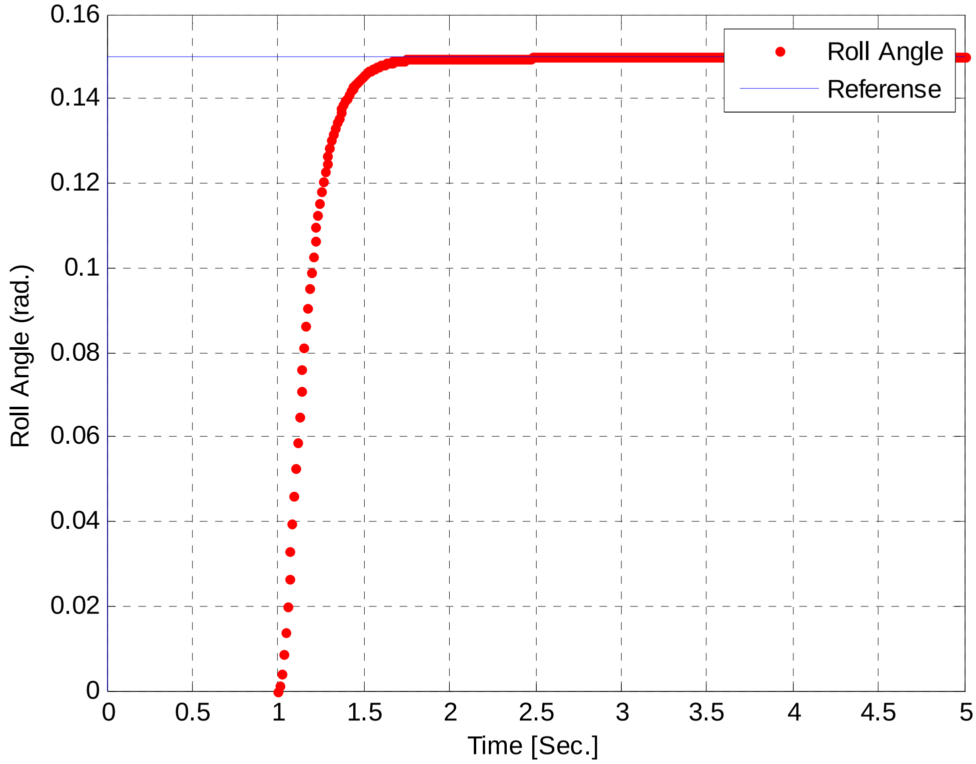

Fig. 6 : Step response of the roll angle for the case of LQR-GA

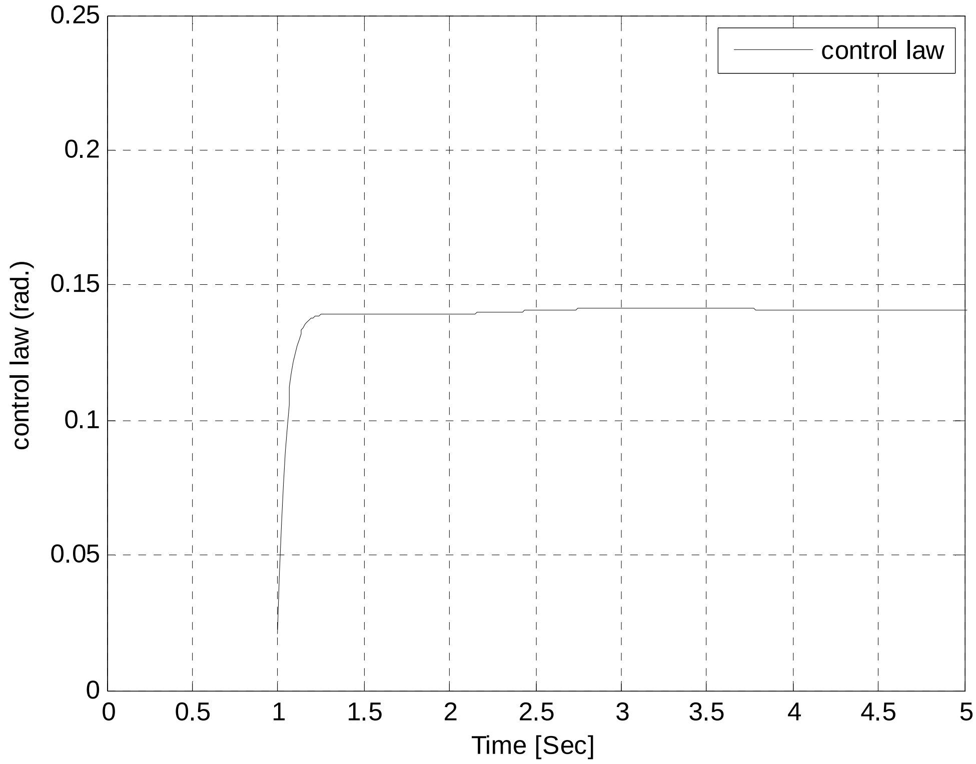

Fig. 7 : The response of the control signal for the case of LQR-GA.

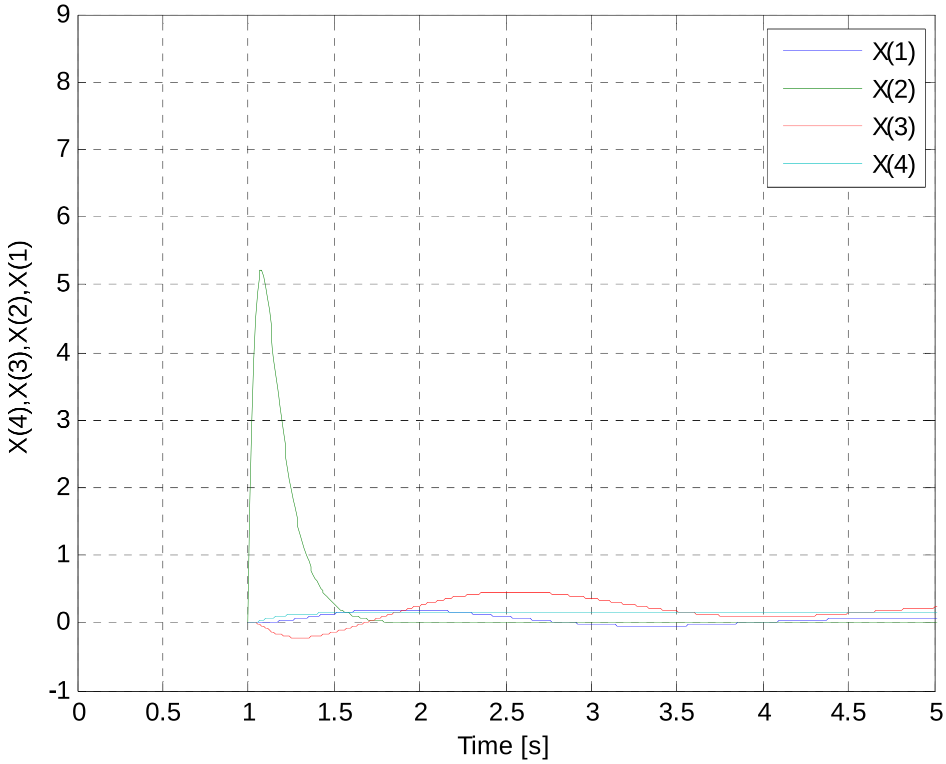

Fig. 8 : The response of aircraft states for the case of LQR-GA.

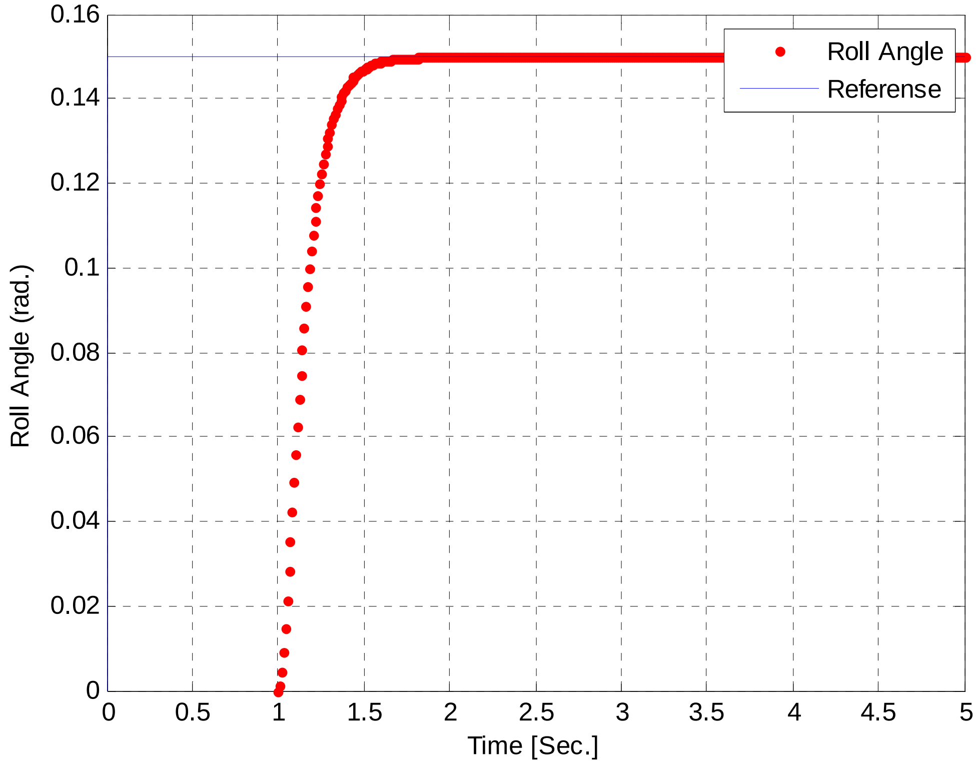

Fig. 9 : Step response of the roll angle for the case of LQR-PSO.

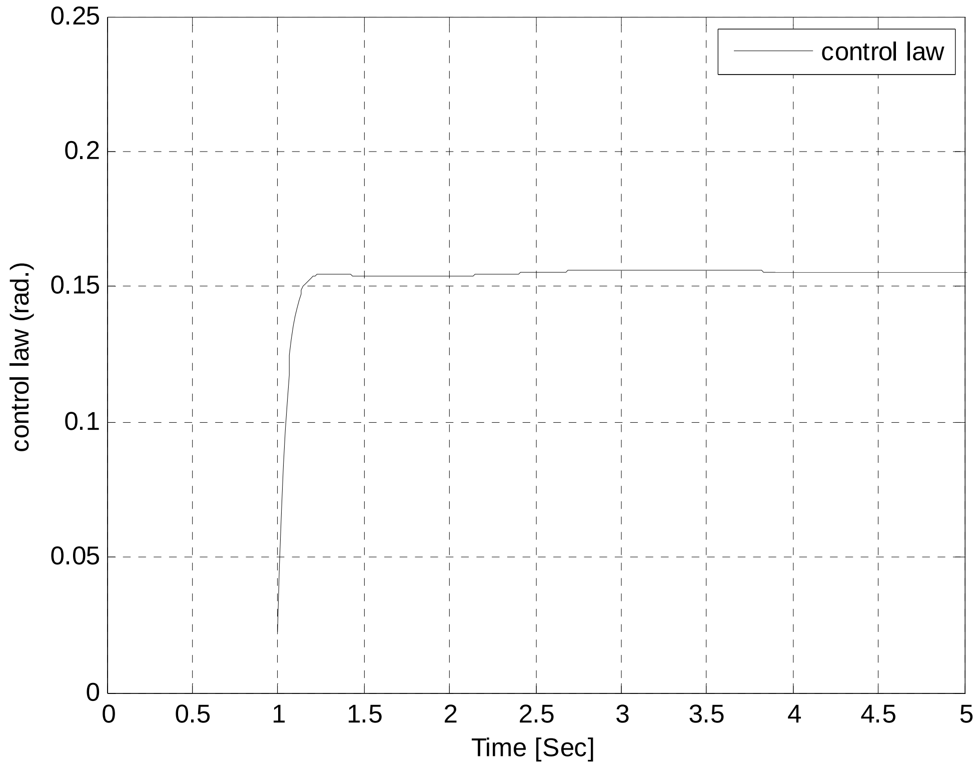

Fig. 10 : The response of the control signal for the case of LQR-PSO

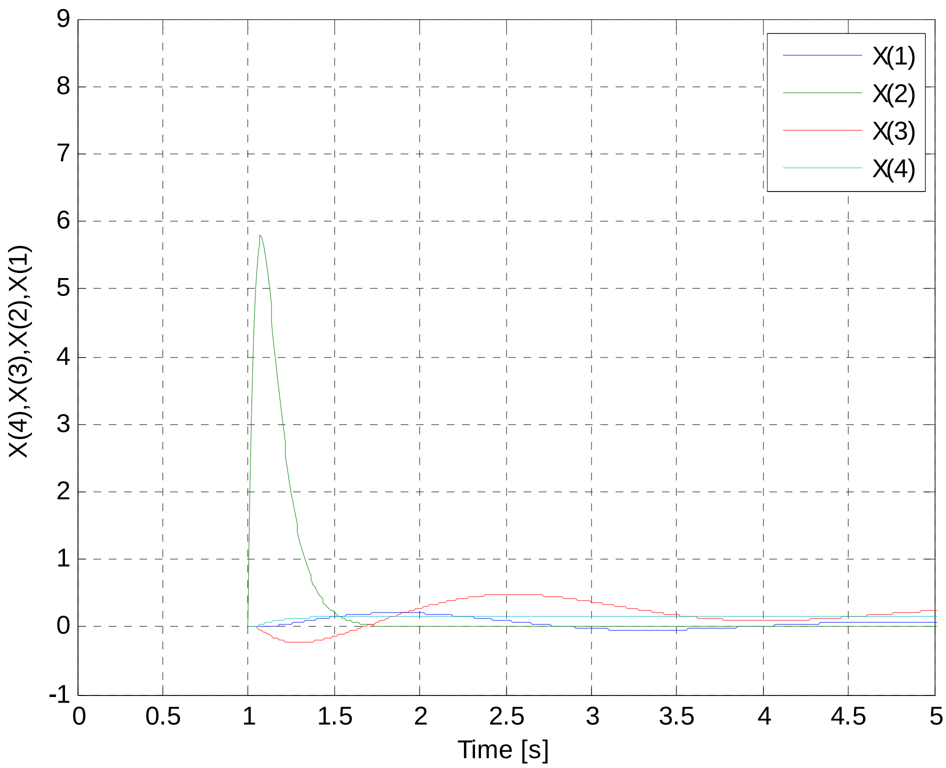

Fig. 11 : The response of aircraft states for the case of LQR-PSO

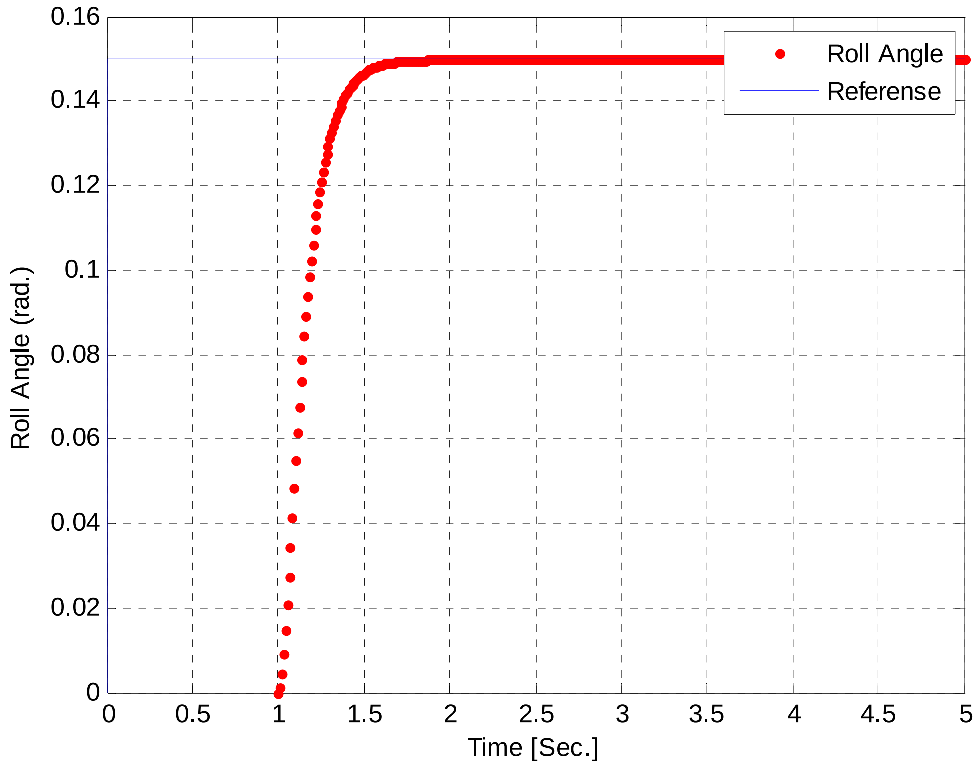

Fig. 12 : Step response of the roll angle for the case of LQR-ABC

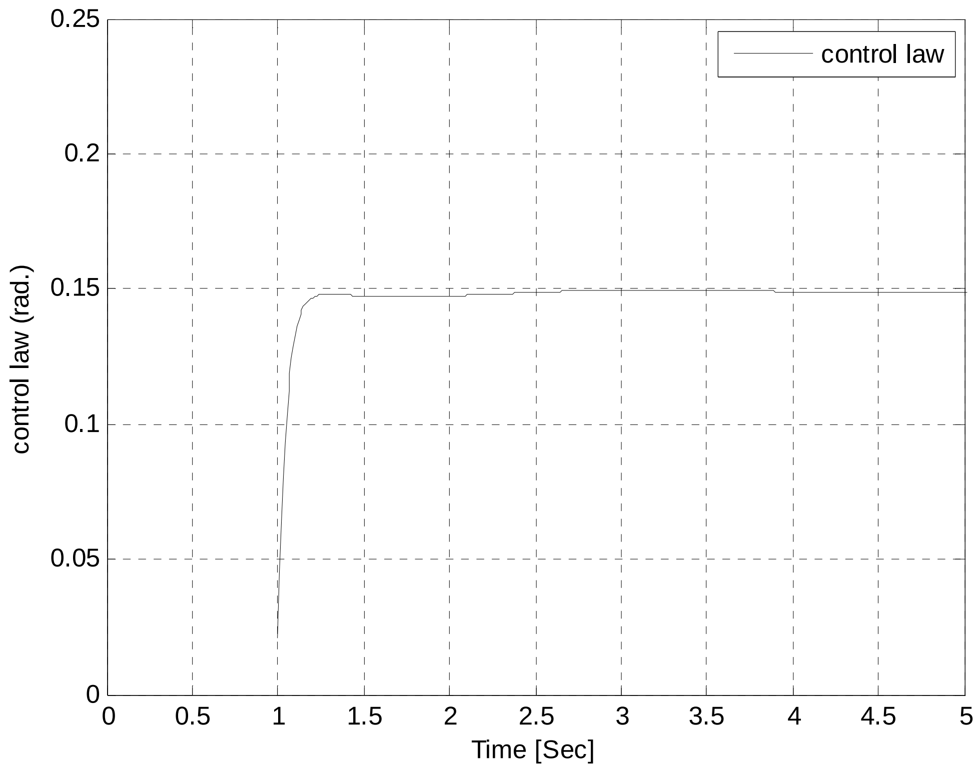

Fig. 13 : The response of the control signal for the case of LQR-ABC

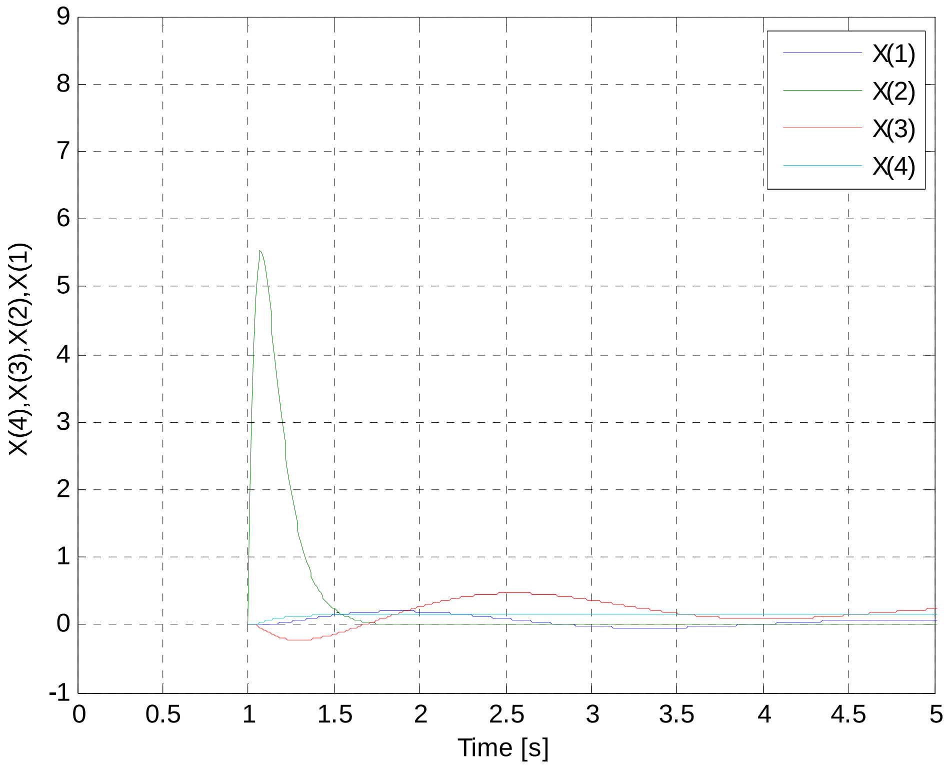

Fig. 14 : The response of aircraft states for the case of LQR-ABC

In order to deeply analyze the closed loop performance of the three proposed design techniques, the derivative of the control signals at the initial transient period are presented in Fig. (15).

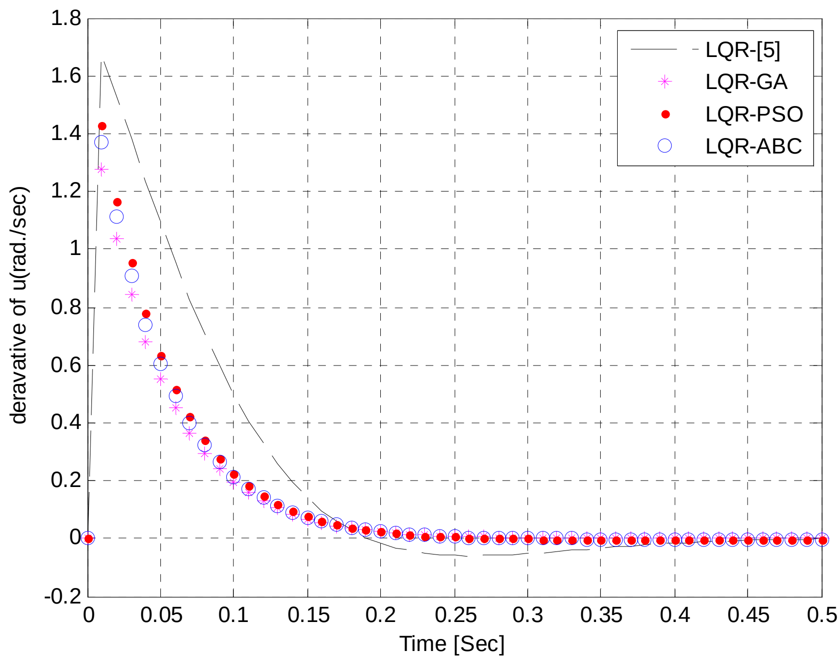

Fig. 15 : The derivative of the control signals at the initial transient period